Parton Pole Matrix Elements and Universality of TMD-fragmentation

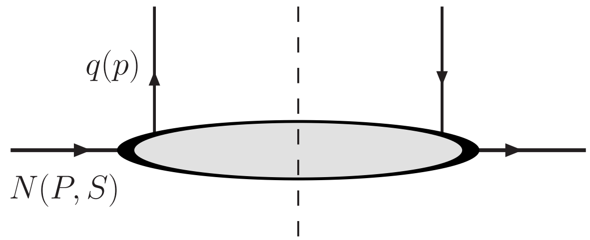
(A. Metz, Temple University, Philadelphia)

- Introduction
- Parton pole matrix elements (PPMEs) for fragmentation
- Universality of TMD-fragmentation
- Summary

mainly based on S. Meißner, A. Metz, arXiv:0812.3783

2- and 3-parton correlators

- 2-parton correlators (quarks PDFs)



$$\Phi^q(x) \sim \int d\xi^- \, e^{ip\cdot\xi} \, \langle P| \, \bar{\psi}(0) \, \Gamma \, \psi(\xi) \, |P\rangle \Big|_{\xi^+=\xi_T=0}$$

- 3-parton correlators

$$\Phi_F^q(x, x') \quad \sim \quad \int d\xi^- d\zeta^- \, e^{ip\cdot\xi} \, e^{i(p'-p)\cdot\zeta}$$
$$\times \, \langle P| \, \bar{\psi}(0) \, \Gamma \, F^{+i}(\zeta) \, \psi(\xi) \, |P\rangle \Big|_{\xi^+=\xi_T=\zeta^+=\zeta_T=0}$$

$\longrightarrow$ 4 independent (leading) functions for $\Gamma = \{\gamma^+, \; \gamma^+\gamma_5, \; i\sigma^{j+}\gamma_5\}$
(Jaffe, Ji, 1992)

$\longrightarrow$ Twist-3 effects

Parton pole matrix elements

- PPMEs: one of the 3 partons has vanishing (longitudinal) momentum
  $\longrightarrow$ e.g., gluon pole matrix element (GPME): $\Phi_F^q(x, x' = x)$

- PPMEs can be used to describe SSAs
  (Efremov, Teryaev, 1982, ... / Qiu, Sterman, 1991, ...)

- Large amount of recent work on PPMEs and SSAs

- Relation to TMDs (Boer, Mulders, Pijlman, 2003)

$$
\begin{aligned}
f_{1T}^{\perp(1)}(x) \quad &\sim \quad \int d^2\vec{p}_T \, \vec{p}_T^{\,2} \, f_{1T}^{\perp}(x, \vec{p}_T^{\,2}) \sim T_F(x, x) \\
h_1^{\perp(1)}(x) \quad &\sim \quad T_F^{(\sigma)}(x, x)
\end{aligned}
$$

$\longrightarrow$ TMD and twist-3 approach to spin/azimuthal asymmetries intimately connected

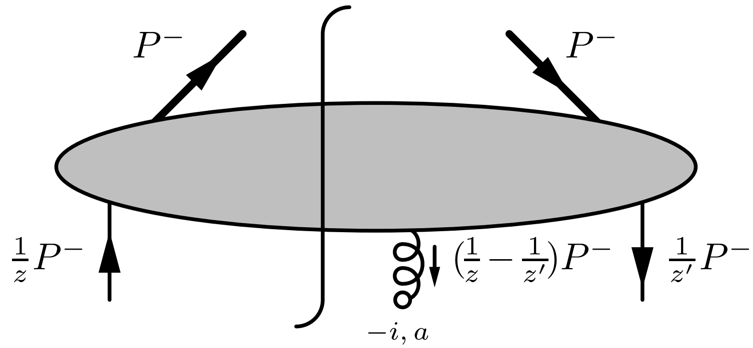
- PPMEs were also used/discussed for parton fragmentation
  (Koike et al., 2001, ... / Boer, Mulders et al., 2003, ...)

- But, mere existence of PPMEs (GPMEs) for fragmentation became unclear
  - Existence of GPMEs related to universality of TMD fragmentation:
    if TMD fragmentation universal than GPMEs for fragmentation vanish
    (Boer, Mulders, Pijlman, 2003)
  - T-odd TMD fragmentation functions universal in SIDIS vs e^+e^- annihilation
    (spectator model analysis)
    (Metz, 2002)
  - TMD fragmentation functions universal (spectator model analysis)
    (Collins, Metz, 2004)
  - Collins function universal in $H_1 H_2 \to \pi \mathrm{jet} X$ (spectator model analysis)
    (Yuan, 2007, 2008)
  - GPMEs vanish in spectator model
    (Gamberg, Mukherjee, Mulders, 2008)
  - Collins function at high k_T universal (fixed order pQCD analysis)
    (Yuan, Zhou, 2009)

- Needed: model independent analysis of PPMEs for fragmentation

PPMEs for fragmentation

- Definition of 3-parton correlators (in light-cone gauge)



$$\Delta_F^i(\frac{1}{z},\frac{1}{z'}) = \sum_X \int \frac{d\xi^+}{2\pi}\frac{d\eta^+}{2\pi}\, e^{i\frac{1}{z'}P^-\xi^+ + i\left(\frac{1}{z}-\frac{1}{z'}\right)P^-\eta^+}$$
$$\times \frac{1}{3}\,\mathrm{Tr}\Big[\langle 0|\, g\, t_a F_a^{-i}(\eta^+)\,\psi(\xi^+)\,|P,X\rangle\langle P,X|\,\bar\psi(0^+)\,|0\rangle\Big]$$

Analogous for $\bar q g \bar q$ and ggg correlator

- PPMEs
  - $\frac{1}{z}=\frac{1}{z'}$ soft gluon pole (GPME)
  - $\frac{1}{z'}=0$ soft fermion pole (FPME)

- Support properties of 3-parton correlators

$\longrightarrow$ Insert complete set of states $|Y\rangle$

$$
\begin{aligned}
\Delta_F^i\Big(\frac{1}{z},\frac{1}{z'}\Big) &= \sum_{X,Y}\int\frac{d\xi^+}{2\pi}\frac{d\eta^+}{2\pi}\,e^{i\frac{1}{z'}P^-\xi^+ + i\left(\frac{1}{z}-\frac{1}{z'}\right)P^-\eta^+} \\
&\quad\times\frac{1}{3}\,\mathrm{Tr}\Big[\langle 0|\,g\,t_a F_a^{-i}(\eta^+)\,|Y\rangle\langle Y|\,\psi(\xi^+)\,|P,X\rangle\,\langle P,X|\,\bar{\psi}(0^+)\,|0\rangle\Big] \\
&= \sum_{X,Y}\delta\Big(\big(\frac{1}{z}-1\big)P^- - \textstyle\sum_i p_i^-\Big)\,\delta\Big(\big(\frac{1}{z}-\frac{1}{z'}\big)P^- - \textstyle\sum_j q_j^-\Big) \\
&\quad\times\frac{1}{3}\,\mathrm{Tr}\Big[\langle 0|\,g\,t_a F_a^{-i}(0^+)\,|Y\rangle\langle Y|\,\psi(0^+)\,|P,X\rangle\,\langle P,X|\,\bar{\psi}(0^+)\,|0\rangle\Big]
\end{aligned}
$$

$\longrightarrow$ One has: $p_i^- \geq 0\,, \quad q_j^- \geq 0$

$\longrightarrow$ This implies:

$$
\frac{1}{z}\geq 1 \quad \text{and} \quad \frac{1}{z}\geq\frac{1}{z'}
$$

$\longrightarrow$ Note: GPMEs vanish as soon as one spectator in $|Y\rangle$ is massive

$\longrightarrow$ Exchange quark and gluon field, and insert complete set of states $|Y\rangle$

$$\Delta^{i}_{F}(\frac{1}{z},\frac{1}{z'}) \;=\; \sum_{X,Y} \delta\Big((\frac{1}{z}-1)P^{-} - \textstyle\sum_i p_i^{-}\Big)\,\delta\Big(\frac{1}{z'}P^{-} - \textstyle\sum_j q_j^{-}\Big)$$
$$\times\frac{1}{3}\,\mathrm{Tr}\Big[\langle 0|\, t_a\, \psi(0^+)\,|Y\rangle\langle Y|\, gF_a^{-i}(0^+)\,|P,X\rangle\,\langle P,X|\,\bar{\psi}(0^+)\,|0\rangle\Big]$$

$\longrightarrow$ This implies:

$$\frac{1}{z} \geq 1 \quad \text{and} \quad \frac{1}{z'} \geq 0$$

$\longrightarrow$ Note: FPMEs vanish as soon as one spectator in $|Y\rangle$ is massive

$\longrightarrow$ What happens in the (academic) case of massless spectators ?

- (Academic) case: all spectators massless

  $\longrightarrow$ Consider the matrix elements

$$
\begin{aligned}
M^{-i}(q_j) &= \langle 0|\, g\, t_a F_a^{-i}(0^+)\,|{\color{red}Y}\rangle \\
& \quad\ \langle 0|\, t_a\, \psi(0^+)\,|{\color{red}Y}\rangle
\end{aligned}
$$

  $\longrightarrow$ Vanishing of second matrix element obvious

  $\longrightarrow$ Decompose first matrix element according to

$$
\begin{aligned}
M^{\mu\nu}(q_j) &= \sum_{m,n} \Big[q_m^\mu q_n^\nu\, A_{mn}(q_j) + q_m^\mu \epsilon^\nu(q_n)\, B_{mn}(q_j) \\
&\quad + \epsilon^\mu(q_m)\, \epsilon^\nu(q_n)\, C_{mn}(q_j) - \{\mu \leftrightarrow \nu\} \Big]
\end{aligned}
$$

  $\longrightarrow$ First matrix element M^{-i} vanishes because of

$$
q_j^- = \vec{q}_{jT} = \epsilon^-(q_j) = 0
$$

- In summary

  $\longrightarrow$ Analysis implies:

$$0 \leq z \leq 1 \quad \text{and} \quad \frac{1}{z} > \frac{1}{z'} > 0$$

  $\longrightarrow$ In other words:

$$\begin{aligned}
\Delta^i_F(\frac{1}{z}, \frac{1}{z}) &= \Delta^i_F(\frac{1}{z}, 0) = 0 \\
\bar{\Delta}^i_F(\frac{1}{z}, \frac{1}{z}) &= \bar{\Delta}^i_F(\frac{1}{z}, 0) = 0 \\
\hat{\Gamma}^{i,jk}_{F,f/d}(\frac{1}{z}, \frac{1}{z}) &= \hat{\Gamma}^{i,jk}_{F,f/d}(\frac{1}{z}, 0) = 0
\end{aligned}$$

  $\longrightarrow$ Result does not exclude PPMEs for parton distributions

Universality of TMD-fragmentation

- Why nontrivial ?

$$\Delta^{[\mathcal{U}]}(\frac{1}{z},\vec{k}_T) = \sum_X \int \frac{d\xi^+}{2\pi}\frac{d^2\vec{\xi}_T}{(2\pi)^2}\,e^{ik\cdot\xi} \times \frac{1}{3}\,\mathrm{Tr}\Big[\langle 0|\,\mathcal{W}^{[\mathcal{U}]}(0,\xi)\,\psi(\xi)\,|P,X\rangle\,\langle P,X|\,\bar{\psi}(0)\,|0\rangle\Big]_{\xi^-=0}$$

  - A priori different Wilson lines (TMDs) in different processes
  - Time-reversal does not give a relation between different definitions

- Why important ?
  - Prerequisite for combined analysis of data from SIDIS and $e^+e^- \longrightarrow H_1H_2X$ (and more complicated processes like $H_1H_2 \longrightarrow \pi\mathrm{jet}X$) (Efremov, Goeke, Schweitzer, 2006, ... / Anselmino et al., 2007, ...)
  - In particular, prerequisite for first extraction of transversity (Anselmino et al., 2007, ...)

- Generality of existing analyzes showing universality of Collins function and other TMD fragmentation functions was doubted for 2 reasons:
  - Spectator models (Note: also used in proof of q_T-integrated Drell-Yan) (Bodwin, 1984 / Collins, Soper, Sterman, 1985, 1988)
  - Low order in perturbation theory

- Zeroth moment of TMD-correlator:

$$\Delta(\frac{1}{z}) = \int d^2\vec{k}_T \, \Delta^{[\mathcal{U}]}(\frac{1}{z}, \vec{k}_T)$$

$\longrightarrow$ Process dependence disappears

$\longrightarrow$ $D_1(z),\ G_1(z),\ H_1(z)$ are universal

- First moment of TMD-correlator:
  (Boer, Mulders, Pijlman, 2003 / Bomhof, Mulders, 2007)

$$\begin{aligned} \Delta_{\partial}^{i[\mathcal{U}]}(\frac{1}{z}) &= \int d^2\vec{k}_T\, k_T^i\, \Delta^{[\mathcal{U}]}(\frac{1}{z},\vec{k}_T) \\ &= \tilde{\Delta}_{\partial}^{i}(\frac{1}{z}) + C_F^{[\mathcal{U}]}\,\pi\Delta_F^{i}(\frac{1}{z},\frac{1}{z}) \end{aligned}$$

$\longrightarrow$ Process dependence contained in calculable gluonic pole factors $C_F^{[\mathcal{U}]}$

$\longrightarrow$ Process dependent part given by GPMEs

$\longrightarrow$ Model-independent analysis of GPMEs shows (in particular) universality of

$$H_1^{\perp(1)}(z) \sim \int d^2\vec{k}_T\, \vec{k}_T^{\,2}\, H_1^{\perp}(z,\vec{k}_T^{\,2})$$

Summary

- PPMEs for fragmentation vanish (model independent proof)

- PPMEs for fragmentation cannot generate SSAs in collinear factorization

- But, other twist-3 collinear fragmentation correlators can well do so (Yuan, Zhon, 2009)

- Model-independent proof of universality of certain k_T-moment of TMD-fragmentation functions

- Analysis may be extended to higher k_T-moments (UV- and other divergences ?)